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SOME ASPECTS OF SURFACE ROUGHNESS MEASUREMENT

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SUMMARY

The connection between the profile and surface Power Spectral Densities of a rough surface is investigated, and explicit functional relations between the two are obtained for isotropic surfaces. These relations allow the surface Power Spectral Density (PSD) to be obtained from the PSD of a single profile, for isotropic surfaces. For anisotropic surfaces, it is shown how the surface PSD may be obtained from the cross-spectra of several parallel profiles. Techniques for obtaining these cross-spectra are briefly mentioned.

A simple example of an isotropic surface is used to show that the profile PSD may seriously distort the spectral content of the surface roughness by giving undue weight to long wavelengths at the expense of short wavelengths.

Questions of filtering of the surface and profile PSDs are discussed for isotropic surfaces, and it is shown that removal of all wavelengths smaller than λ_0 on the surface requires their removal on the profile, but in addition requires some attenuation of all wavelengths on the profile greater than λ_0. The question of which profile filters are admissible in the sense that they give rise to physically realizable surface filters (with $0 \leqslant$ attenuation $\leqslant 1$) is also examined, and it is shown that all profile filters involving an infinitely sharp cut-off at some wavelength are inadmissible.

Based on an examination of the connection between the surface PSD and various surface statistics of interest, four indices of anisotropy involving the moments of this PSD are developed. It is shown how these indices may be evaluated by means of measurements on five nonparallel profiles.

1. INTRODUCTION

In recent years, random process theory has begun to find an increasingly important place in the characterization of the surfaces of solids[1–4]. It is now accepted that solid surfaces must be considered to be two-dimensional random processes, though detailed analyses of such processes have so far been restricted to the case where the surface in question can be assumed to be isotropic and Gaussian[1,3].

On the other hand, detailed experimental observations of solid surfaces have been restricted to examinations of profiles, the exception being the excellent "micro-

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cartography" of Williamson and his colleagues[5,6]. Microcartography, however, is a fairly sophisticated and expensive technique, and does not appear likely to become a practical tool for the examination of surfaces. It thus appears necessary to examine in an analytical fashion the relation between profile and surface characteristics. Some important statistics of surfaces have been examined[1], and a detailed comparison made with profile statistics, for isotropic, Gaussian surfaces. The purpose of this paper is to examine the connection between the Power Spectral Density (PSD) of the surface and that of a profile, for isotropic (but not necessarily Gaussian) surfaces, with a view to determining how the profile PSD is to be interpreted. A further purpose is the development of analytical techniques for the study of anisotropic surfaces.

2. ANALYSIS

2.1. *Relation between surface and profile PSDs*

Consider an isotropic surface whose height is $z(x_1, x_2)$, x_1 and x_2 being Cartesian coordinates in the mean plane of the surface. We may then define an autocorrelation function by[1]

$$R(r) = \lim_{\substack{L_1 \to \infty \\ L_2 \to \infty}} \frac{1}{4L_1 L_2} \int_{-L_1}^{L_1} \int_{-L_2}^{L_2} z(x_1, x_2)\, z(x_1+x_{10}, x_2+x_{20})\, \mathrm{d}x_1 \mathrm{d}x_2 \,, \tag{1}$$

where

$$r = (x_{10}^2 + x_{20}^2)^{\frac{1}{2}} \,. \tag{2}$$

Since the surface is isotropic, the two point autocorrelation function depends only on the distance from one point to the other, and not the direction. Furthermore, the same autocorrelation function would be obtained for surface heights measured on a profile, and it is therefore the function that bridges the gap between surface and profile characteristics.

If $\Phi^P(k_1)$ be the profile PSD, k_1 being the wavenumber along the profile ($k_1 = 2\pi/\lambda$, λ = wavelength), then[1]

$$\Phi^P(k_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(r) \exp(-ik_1 r) \mathrm{d}r \,. \tag{3}$$

Similarly, if $\Phi^s(k_1, k_2)$ be the surface PSD,

$$\Phi^s(k_1, k_2) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(x_1, x_2) \exp[-\mathrm{i}(k_1 x_1 + k_2 x_2)] \,\mathrm{d}x_1 \mathrm{d}x_2 \,. \tag{4}$$

For an isotropic surface, $R(x_1, x_2)$ being a function only of $(x_1^2 + x_2^2)$, eqn. (4) reduces to

$$\Phi^s(k) = \frac{1}{2\pi} \int_0^{\infty} rJ_0(kr)\, R(r) \mathrm{d}r \,, \tag{5}$$

where J_0 is the Bessel function of order zero, and

$$k^2 = k_1^2 + k_2^2 \,. \tag{6}$$

The inverse relationships corresponding to eqns. (3) and (5) are:

$$R(r) = \int_{-\infty}^{\infty} \Phi^P(k_1) \exp(ik_1 r) \mathrm{d}k_1 \tag{7}$$

and

$$R(r) = 2\pi \int_0^\infty kJ_0(kr)\,\Phi^s(k)\,dk\,. \tag{8}$$

It is clear that there is a unique relation between Φ^P and Φ^s, determined by the autocorrelation function $R(r)$. In fact, by combining eqns. (3) and (8), we obtain,

$$\Phi^P(k_1) = 2\int_{k_1}^\infty \frac{k\Phi^s(k)\,dk}{(k^2-k_1^2)^{\frac{1}{2}}}\,. \tag{9}$$

The relation inverse to eqn. (9) is obtained by noting that $\Phi^P(k_1)$ is the Abel transform of $\Phi^s(k)$[7]. The inverse is known in general form:

$$\Phi^s(k) = \frac{1}{\pi}\int_k^\infty (k_1^2-k^2)^{\frac{1}{2}} \left\{\frac{\Phi''^P(k_1)}{k_1} - \frac{\Phi'^P(k_1)}{k_1^2}\right\} dk_1\,, \tag{10}$$

where the primes denote differentiation with respect to k_1.

Two important issues require examination:

(1) Whether the profile PSD may be misleading as to the dominant wavelengths on the surface, and

(2) How filtering of the surface PSD is related to filtering of the profile PSD. Filtering out of short wavelengths has been shown to be of some importance in the contact of rough surfaces[2,3].

2.2. *Distortion of the surface by the profile PSD*

For the sake of clarity (but without loss of relevance), consider a specific autocorrelation function[2], one that is easy to manipulate mathematically, even though it has been shown to be unrealistic[1]. Specifically, let

$$R(r) = \sigma^2 e^{-\beta|r|}\,, \tag{11}$$

where σ is the r.m.s. height of the surface. Introducing this into eqns. (3) and (4), we obtain

$$\Phi^P(k_1) = \frac{\sigma^2\beta}{\pi(\beta^2+k_1^2)}\,, \tag{12}$$

and

$$\Phi^s(k) = \frac{\sigma^2\beta}{2\pi(\beta^2+k^2)^{\frac{3}{2}}}\,. \tag{13}$$

In order to assess the importance of different wavenumbers (*i.e.*, inverse wavelengths), consider the magnitude of these two PSD's lying in the range $(|k_0|, |k_0+dk_0|)$. Since $\Phi^P(-k_1) = \Phi^P(k_1)$, we obtain from eqn. (12),

$$\Delta\Phi^P = \frac{2\sigma^2\beta\,dk_0}{\pi(\beta^2+k_0^2)}\,. \tag{14}$$

The surface PSD is a function of k but independent of the polar angle $\theta = \tan^{-1}(k_2/k_1)$; integrating over θ, we obtain from eqn. (13),

$$\Delta\Phi^s = \frac{\sigma^2\beta k_0\,dk_0}{(\beta^2+k_0^2)^{\frac{3}{2}}}\,. \tag{15}$$

A comparison of eqns. (14) and (15) reveals a significant difference between the profile and surface PSD's. Whereas the profile PSD claims that the contribution to the surface roughness from wavelengths λ such that $\lambda \gg 2\pi/\beta$ is fairly uniform, eventually decreasing as λ decreases, the surface PSD claims that there is a somewhat dominant wavelength, given by $\lambda = 2\pi(2)^{\frac{1}{2}}/\beta$, where $\Delta\Phi^s$ has a maximum. In general, we find, from eqns. (14) and (15), that

$$\frac{\Delta\Phi^s}{\Delta\Phi^P} = \frac{\pi k_0}{2(\beta^2 + k_0^2)^{\frac{1}{2}}} \begin{cases} \rightarrow \pi/2 \text{ as } k_0 \rightarrow \infty, \\ \rightarrow 0 \text{ as } k_0 \rightarrow 0. \end{cases} \quad (16)$$

Thus it appears that the profile distorts the surface in such a way as to give undue weight to long wavelengths, at the expense of short wavelengths.

2.3. *Filtering of the surface PSD*

Consider what happens by filtering of the surface PSD so as to remove short wavelengths (*i.e.*, large k). Filtering in the general case amounts to multiplying the surface PSD by a surface filter function $F^s(k)$ such that $0 \leqslant F^s \leqslant 1$. Thus the filtered surface PSD is

$$\Phi_f^s(k) = \Phi^s(k) F^s(k). \quad (17)$$

Introducing this into eqn. (9), we obtain the filtered profile PSD:

$$\Phi_f^P(k_1) = 2 \int_{k_1}^{\infty} \frac{k\Phi^s(k) F(k) \mathrm{d}k}{(k^2 - k_1^2)^{\frac{1}{2}}}. \quad (18)$$

As a specific example, consider the PSD's given by eqns. (12) and (13) and a filter function $F(k)$ given by

$$F^s(k) = \begin{cases} 1, & |k| \leqslant |k_0| \\ 0, & |k| > |k_0|. \end{cases} \quad (19)$$

Thus, all wavelengths $\lambda < 2\pi/k_0$ are assumed to be removed from the surface. Thus, "What happens to the profile PSD?". Combining eqns. (13) and (17)–(19), we find

$$\Phi_f^P(k_1) = \begin{cases} \dfrac{\sigma^2\beta}{\pi(\beta^2 + k_1^2)} \left(\dfrac{k_0^2 - k_1^2}{\beta^2 + k_0^2}\right)^{\frac{1}{2}}, & |k_1| \leqslant k_0, \\ 0, & |k_1| > k_0. \end{cases} \quad (20)$$

By dividing this expression by the unfiltered profile PSD given in eqn. (12), we obtain the profile filter function $F^P(k_1)$ that is equivalent to the surface filter function $F^s(k)$:

$$F^P(k_1) = \begin{cases} \left(\dfrac{k_0^2 - k_1^2}{\beta^2 + k_0^2}\right)^{\frac{1}{2}}, & |k_1| \leqslant |k_0|, \\ 0, & |k_1| > |k_0|. \end{cases} \quad (21)$$

It is clear from an examination of this expression that while removal of all wavenumbers greater than k_0 in the surface PSD requires their removal in the profile PSD, it additionally requires some attenuation of all other wavenumbers.

2.4. *Filtering of the profile PSD*

Finally, consider what happens to the surface PSD when the profile PSD is filtered by removal of all wavelengths greater than k_0. The filtered profile PSD is given by

$$\Phi_f^P(k_1) = \begin{cases} \dfrac{\sigma^2 \beta}{\pi(\beta^2+k_1^2)}, & |k_1| \leqslant |k_0| \\ 0 & , \quad |k_1| > |k_0|. \end{cases}$$

To determine the filtered surface PSD, one now has to use eqn. (10).

By considering $\Phi_f'^P$ to have a δ-function singularity at $k_1 = k_0$, it may be shown that

$$\Phi_f^s(k) = \frac{8\sigma^2\beta}{\pi^2}\int_k^{k_0} \frac{k_1(k_1^2-k^2)^{\frac{1}{2}}\,\mathrm{d}k_1}{(\beta^2+k_1^2)^3} + \frac{\sigma^2\beta}{\pi^2(\beta^2+k_0^2)} \times \begin{cases} (k_0^2-k^2)^{-\frac{1}{2}} + \dfrac{2(k_0^2-k^2)^{\frac{1}{2}}}{\beta^2+k_0^2}, & k \leqslant k_0 \\ 0 & , \quad k > k_0 \end{cases}$$

An examination of this expression shows that there is a singularity at $k = k_0$ in the filtered surface PSD, whereas none existed in the unfiltered surface PSD. Thus, the manner of profile filtering used results in an increase in the surface PSD at certain wavenumbers, which is physically inadmissible. It may be shown quite generally that in this sense, an infinitely sharp cut-off in the profile filter is inadmissible.

3. A SAND-HEAP ANALOGY

All of the preceding results are explained by a sand-heap analogy, in which the surface PSD $\Phi^s(k_1, k_2)$ is considered to be a sand-heap on the (k_1, k_2) plane. The profile PSD for a profile in an arbitrary direction is obtained by collapsing all the sand onto the profile, along lines perpendicular to the profile. This analogy may be shown to be rigorously true[1] for arbitrary surfaces (non-isotropic, non-Gaussian, etc.). The foundations of the analogy lie in the fact that what appears in a profile (assumed parallel to the x_1-axis) to be a wave of wavelength λ_0 may, on the surface, be a wave of any wavelength $\lambda \leqslant \lambda_0$ "propagating" in a direction θ such that $\lambda_0 = \lambda/\cos\theta$. Thus, a variety of waves on the surface, of differing wavelengths and directions, contribute to the profile PSD at $k = 2\pi/\lambda$.

This analogy clearly shows, for example, that when the sand beyond a radius k_0 is removed, the amount of sand falling on the k_1-axis is reduced for all values of k_1, right down to zero.

4. ANISOTROPIC SURFACES

For the limiting anisotropic case, namely two-dimensional surfaces, it is clear that the surface and profile PSDs are identical. For surfaces that are neither isotropic nor two-dimensional, the surface PSD may be obtained by examining the cross-spectra of parallel profiles.

4.1. *Definition of the cross-spectrum*

Consider two profiles parallel to the x_1-axis, lying at $x_2=\xi$ and at $x_2=x_{20}+\xi$. The cross-correlation between these two profiles is defined as

$$R^c(x_{10};\xi,x_{20})=\operatorname*{Lim}_{L\to\infty}\frac{1}{2L}\int_{-L}^{L}z(x_1,\xi)(x_1+x_{10},\xi+x_{20})\mathrm{d}x_1\,. \tag{22}$$

The cross-spectrum is then defined in analogy to eqn. (3):

$$\Phi^c(k_1;\xi,x_{20})=\frac{1}{2\pi}\int_{-\infty}^{\infty}R^c(x_{10};\xi,x_{20})\exp(-ik_1x_{10})\mathrm{d}x_{10}\,. \tag{23}$$

Clearly, the profile PSD of a profile at $x_2=\xi$ is found by setting $x_{20}=0$ in Φ^c:

$$\Phi^P(k_1;\xi)=\Phi^c(k_1;\xi,0)\,. \tag{24}$$

Assume that the surface is homogeneous, then the profile PSD should not depend on ξ (this is in fact a definition of homogeneity). Thus, we must have

$$\Phi^c(k_1;\xi,x_{20})=\Phi^c(k_1;x_{20})\,. \tag{25}$$

By tracing backwards to eqn. (22), we see that for homogeneous surfaces we must have

$$R^c(x_{10};\xi,x_{20})=R^c(x_{10};x_{20}). \tag{26}$$

Now the general autocorrelation function, defined by

$$R(x_{10},x_{20})=\operatorname*{Lim}_{\substack{L_1\to\infty\\L_2\to\infty}}\frac{1}{4L_1L_2}\int_{-L_1}^{L_1}\int_{-L_2}^{L_2}z(x_1,x_2)z(x_1+x_{10},x_2+x_{20})\mathrm{d}x_1\mathrm{d}x_2\,,$$

is clearly obtained by averaging the expressions in eqn. (22) over ξ. Since these expressions are independent of ξ for homogeneous surfaces, we find

$$R^c(x_{10};x_{20})=R(x_{10},x_{20})\,. \tag{27}$$

From eqn. (4) the surface PSD is the two-dimensional Fourier transform of $R(x_{10},x_{20})$. On the other hand, the cross-spectrum defined in eqn. (23) is seen from eqns. (26) and (27) to be the one-dimensional Fourier transform of $R(x_{10},x_{20})$. Thus, the surface PSD is obtained by further transforming the cross-spectrum $\Phi^c(k_1;x_{20})$:

$$\begin{aligned}\Phi^s(k_1.\,k_2)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi^c(k_1;x_{20})\exp(-ik_2x_{20})\mathrm{d}x_{20}\\&=\frac{1}{\pi}\int_{0}^{\infty}\Phi^c(k_1;x_{20})\cos(k_2x_{20})\mathrm{d}x_{20}\,.\end{aligned} \tag{28}$$

4.2. *Obtaining the cross-spectrum*

A technique for obtaining the cross-spectrum has been discussed by Gray and Johnson[8]. The basic requirement is a series of parallel profiles with a common origin. Gray and Johnson meet this requirement by using two rigidly connected inductance probes traversing the surface. An alternative is to use the technique of relocation profilometry developed by Williamson and his colleagues[9], in which the common origin is determined by using a digital computer to maximize $R^c(0,x_{20})$.

In either case, two signals are obtained for two profiles, that may be recorded on tape. Well known techniques[10] may then be used to obtain the cross-spectrum, involving procedures identical to those used in obtaining the PSD of a single profile. If a number of parallel profiles are obtained, a smoothed approximation to the cross-spectrum as a function of the profile separation x_{20} may be developed. The surface PSD is then obtained by using eqn. (28), the computations in which may also be implemented on a digital computer.

4.3. *Measures of anisotropy*

It has been shown how to obtain the surface PSD of an anisotropic surface. Equally important (particularly in a study of surface typology) is whether it is possible to define some simple indices of anisotropy, which are easy to measure. An approach to the development of such indices based on the surface PSD, and particularly on those characteristics of the surface PSD that are likely to arise in a determination of important surface statistics is presented.

It has been shown that for Gaussian surfaces, only seven parameters are necessary in order to obtain surface statistics such as the density of summits, their height distribution and their tip curvatures [1,11]. These characteristic parameters, or "invariants" of the surface are[1,11].

$$I_1 = m_{00}$$
$$I_2 = (m_{02} + m_{20})$$
$$I_3 = (m_{20}m_{02} - m_{11}^2)$$
$$I_4 = (m_{40} + 2m_{22} + m_{04})$$
$$I_5 = 2(m_{40}m_{04} - 4m_{13}m_{31} + 3m_{22}^2)$$
$$I_6 = (m_{40} + m_{22})(m_{04} + m_{22}) - (m_{31} + m_{13})^2$$

and

$$I_7 = m_{40}(m_{22}m_{04} - m_{13}^2) - m_{31}(m_{31}m_{04} - m_{13}m_{22}) + m_{22}(m_{13}m_{31} - m_{22}^2), \quad (29)$$

where the m's are moments of the surface PSD defined by

$$m_{ij} = \iint_{-\infty}^{\infty} k_1^i k_2^j \Phi^s(k_1, k_2) \mathrm{d}k_1 \mathrm{d}k_2 . \quad (30)$$

The quantities in eqn. (29) are invariants in the sense that they do not depend on the orientation of the (x_1, x_2) axes [this orientation influences the functional form of $\Phi^s(k_1, k_2)$].

Longuet-Higgins[12], in a discussion of these invariants, gives them the following interpretation. I_1 is simply the mean-square height of the surface; I_2 the mean-square gradient. The quantity γ defined by

$$\gamma^2 = \frac{I_2 - (I_2^2 - 4I_3)^{\frac{1}{2}}}{I_2 + (I_2^2 - 4I_3)^{\frac{1}{2}}}, \quad (31)$$

is a measure of the mean-square angular deviation of the energy in the surface PSD from a certain direction, termed a "principal direction". More importantly, it is possible to show that for an isotropic surface (on which there are no waves with long crests), $\gamma = 1$; for a two-dimensional surface (on which all the waves have infinitely long crests), $\gamma=0$. Thus $(1/\gamma)$ represents the "long-crestedness" of the surface roughness.

The invariants I_4 and I_5 are measures of the distribution of the total curvature of the surface. At any point on a surface, there are two principal curvatures, say κ_A, κ_B. The product of these two, $\Omega=\kappa_A\kappa_B$, is termed the total curvature. For surfaces with small mean gradient, I_4 is the mean value of Ω, and I_5 is its mean-square value.

The invariant I_6 has not as yet been given a physical interpretation; however, the mean value of Ω^3 is given by $6I_7$. Thus $\beta=\bar{\Omega}_3/(\bar{\Omega}^2)^{\frac{3}{2}}=6I_7/(I_5/2)^{\frac{3}{2}}$ is the skewness of the distribution of Ω.

It may additionally be shown[12] that for an isotropic surface, the seven invariants reduce to only three, as a consequence of the following relations:

$$\begin{aligned} I_3&=\tfrac{1}{4}I_2^2\\ I_5&=\tfrac{3}{8}I_4^2\\ I_6&=\tfrac{1}{4}I_4^2\\ I_7&=\tfrac{1}{64}I_4^3. \end{aligned} \tag{32}$$

The second and fourth of these relations indicate that for an isotropic surface, the skewness is $\beta=2/\sqrt{3}$. We may thus define four invariant parameters whose value is 1 for isotropic surfaces and different from 1 for anisotropic surfaces:

$$p_1=4I_3/I_2^2,$$
$$p_2=8I_5/3I_4^2=\tfrac{8}{3}\times\bar{\Omega}^2/(\bar{\Omega})^2,$$
$$p_3=4I_6/I_4^2,$$

and

$$p_4=3^{\frac{1}{2}}\beta/2=6^{\frac{3}{2}}I_7/I_5^{\frac{3}{2}}. \tag{33}$$

Clearly, variations of these parameters from unity are an adequate characterization of anisotropy for Gaussian surfaces. It appears worth investigating their extrapolation to non-Gaussian surfaces. To conduct such an investigation, it is necessary to develop simple techniques for obtaining the seven invariants and this is possible from measurements on five nonparallel profiles on the surface.

First, the moments of the PSD of a profile at an angle θ from the arbitrarily chosen x_1-axis are defined:

$$m_{n\theta}=\int_{-\infty}^{\infty}k^n\,\Phi_\theta^p(k)\,\mathrm{d}k\,, \tag{34}$$

where $\Phi_\theta^p(k)$ is the PSD of the θ-profile.

It is then possible to show that for an arbitrary surface, the following relations hold between the moments of a profile PSD and those of the surface PSD[1,11]:

$$m_{n\theta}=\sum_{j=0}^{n}m_{n-j,j}C_j^n(\cos\theta)^{n-j}(\sin\theta)^j, \tag{35}$$

where

$$C_j^n=\frac{n!}{j!(n-j)!}\,. \tag{36}$$

Consider five profiles at $\theta=0$, $\pi/6$, $\pi/4$, $\pi/2$ and $3\pi/4$, and designate the profile PSD moments by superscripts 1 ($\theta=0$), 2 ($\theta=\pi/6$), 3 ($\theta=\pi/4$), 4 ($\theta=\pi/2$) and 5 ($\theta=3\pi/4$).

Five sets of equations for the profile moments m_0^i, m_2^i and m_4^i in terms of the surface moments may be written. These equations, when solved, yield:

$$m_{00}=m_0^1,\ m_{20}=m_2^1,\ m_{02}=m_2^4,\ m_{40}=m_4^1,\ m_{04}=m_4^4\,,$$
$$m_{22}=\tfrac{1}{3}(m_4^3+m_4^5)-\tfrac{1}{6}(m_4^1+m_4^4)\,,$$
$$m_{31}=1/4\cdot 3^{\frac{1}{2}}\{-3m_4^1+8m_4^2-(3+3^{\frac{1}{2}})\,m_4^3+m_4^4-(3-3^{\frac{1}{2}})m_4^5\}\,,$$

and

$$m_{13}=1/4\cdot 3^{\frac{1}{2}}\{3m_4^1-8m_4^2+3(1+3^{\frac{1}{2}})m_4^3-m_4^4-3(3^{\frac{1}{2}}-1)m_4^5\}\,. \tag{37}$$

These values of the surface moments, when substituted into eqns. (29) yield the seven invariants; the anisotropy parameters are then obtained from eqns. (33).

A systematic search could yield a set of profile orientations yielding simpler relations between the surface and profile moments than those obtained above.

The profile moments m_0^i, m_2^i and m_4^i are respectively the mean-square height, slope and second derivative of the ith profile; parameters easy to obtain. If, in particular, the profile is Gaussian (in the sense that the height, slope and second derivative have a Gaussian joint-probability distribution), then it may be shown[1,11] that

$$m_2^i=\pi^2 m_0^i\,(D_{\text{zero}}^i)^2$$

and

$$m_4^i=\pi^4 m_0^i\,(D_{\text{zero}}^i)^2\,(D_{\text{extrema}}^i)^2\,. \tag{38}$$

Here, D_{zero}^i is the density of zeroes for the ith profile, and D_{extrema}^i the density of extrema (maxima and minima). Note that the density of extrema of the profile is equal to the density of zeroes of the first derivative of the profile.

5. CONCLUSIONS

To gain a physical picture of the surface, it is important to examine the differences between the surface and profile PSDs. In particular, a partition of the surface roughness into a "dominant" or "broad scale" structure and a superposed fine structure[2] based on the profile PSD can be quite misleading, except for two-dimensional surfaces. For three-dimensional surfaces, the use of one-dimensional random process models has severe limitations.

For any surface, the characteristic function linking the surface and profile PSDs is the autocorrelation function of the surface. For isotropic surfaces, it is possible to derive relatively simple functional relations between the surface PSD and the PSD of an arbitrary profile. These relations show that, in general, the profile PSD distorts the picture of the surface by giving the appearance of a larger long-wavelength roughness content and a smaller short-wavelength content than exist on the surface.

When filtering the profile PSD, it is necessary to consider whether the filter is realistic, in the sense that the corresponding filter function for the surface PSD has a value lying between zero and one. In this sense, all profile filters yielding a sharp cut-off at some wave-number are unrealistic. A preferable approach appears to be to specify surface filter functions, and to then determine the corresponding profile filter. This cannot be done in general terms, but qualitatively, it appears that removal of all wavenumbers on the surface beyond $k=k_0$ requires their removal on the profile, but in addition requires the attenuation to some degree of all wavenumbers on the profile lying between zero and k_0.

A relatively straightforward technique for obtaining the surface PSD of anisotropic surfaces has been presented and appears worth further exploration. The four indices of anisotropy developed are directly related to various surface statistics of interest, as well as being relatively straightforward to obtain, and are also worth exploration.

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